Selection of Parameters for Ant Colony Optimisation Applied to the Optimal Design of Water Distribution Systems

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Abstract: Evolutionary methods, and in particular genetic algorithm optimisation, have been applied successfully to the optimisation of the design and operation of water distribution systems. A relatively new optimisation algorithm method, that is in the same class of evolutionary-type technique, is based on an ant colony metaphor and is known as Ant Colony Optimisation (ACO). ACO uses artificial ants as an optimisation tool. This technique has been successfully applied to the Travelling Salesman Problem (TSP) and has been shown to be more effective than genetic algorithm optimisation. A formulation for Ant Colony Optimisation applied to water distribution system optimisation has been developed previously by the authors. The formulation of the optimisation problem based on Ant Colony Optimisation for optimising pipe network design is based on ants traversing the pipe network with the objective of minimising the cost of the network. There are 6 ACO parameters that need to be selected: the number of ants in a population m, a parameter for controlling the relative importance of pheromone τ for the pipe choice probability α, a parameter for controlling the relative importance of local heuristic factor η for the pipe choice probability β, the pheromone persistence coefficient ρ, the pheromone reward factor R and the pheromone penalty factor P_{pher}. A case study network comprised of 14-pipes has been optimised using the ACO for different combinations of parameters. The ACO technique successfully finds the global optimum solution for the case study problem. Optimal ranges for the six parameters are recommended as an outcome of these investigations (α =3.0 to 5.0, β = 3.0, m = 50 to 800, ρ = 0.8, various combinations of R and P).

Keywords: Ant colony optimisation algorithm; Optimisation; Water distribution systems; Genetic algorithm; Evolutionary method

1. INTRODUCTION

Water distribution networks represent one of the largest infrastructure assets of industrial society. Their construction and maintenance costs millions of dollars. Due to the substantial cost in constructing and operating a water distribution system, optimisation methods have been developed over recent years primarily for cost-minimisation. Techniques for the optimisation of water distribution systems are not new. In fact, many papers have appeared on the optimisation of water distribution systems in the research literature since the mid-1960s. The main techniques that have been used in the past include linear programming, non-linear programming, pruned enumeration and dynamic programming. However, despite the availability of a number of papers, there was up until the early to mid-1990s virtually no transfer from the world of academia to professional practice of the techniques developed in these technical publications. Optimisation of water distribution systems using genetic algorithms (GAs) was first presented by Murphy and Simpson [1992]. Since the mid-1990s, genetic algorithm optimisation [Simpson et al., 1994] and other evolutionary type methods have been applied successfully to the optimisation of water distribution systems. Studies have shown that GAs consistently out-perform other traditional optimisation techniques when applied to water distribution system optimisation. Commercial consulting services using genetic algorithm optimisation software and commercial software are beginning to be more commonly used.

A new optimisation algorithm within the family of evolutionary techniques that is based on an ant colony metaphor, known as Ant Colony

Optimisation (ACO), has been introduced [Dorigo et al., 1996]. Utilising positive feedback via pheromone trails, the ACO algorithm encourages an autocatalytic convergence in the search for optimum solutions where the more ants that follow a trail, the more attractive that trail becomes for being followed. As ants search for a path from the nest to the food source, pheromone is deposited along the path for following ants to detect. Ants are able to locate the path that has a higher concentration of pheromone laid by previous ants. This evolutionary-type technique has been successfully applied to the travelling salesman problem (TSP), quadratic assignment problem [Dorigo et al., 2000], water distribution system optimisation [Foong et al., 2000, Maier et al., 2001], control of robotic vehicles in air combat missions [Sauter et al., 2001], estimation of unsaturated soil hydraulic parameters [Abbaspour et al., 2001] and a range of other combinatorial optimisation problems [Stützle and Hoos, 2000].

The main focus of this paper is to investigate the sensitivity of the ACO method to the selection of parameters. The parameters associated with ACO are based on pheromone deposition, pheromone evaporation and visibility (which depends on the cost of possible pipe paths). Results of a comprehensive parametric study for the ACO method are presented to assess the sensitivity of the ACO method to selection of these parameters.

2. OPTIMAL DESIGN OF PIPE NETWORKS USING ANT COLONY OPTIMISATION

ACO is based on the concept of artificial ants being able to find the shortest route from a nest to a food source by laying down pheromone trails. Unlike natural ants that are effectively blind, artificial ants within an ACO are also given "visibility" or the ability to see what the consequence of their choice may be. Maier et al. [2001] presented a formulation for applying ACO to the optimisation of water distribution systems with a fixed layout. In this type of problem, the shortest path is not an issue but rather selection of the lowest cost combination of pipes. The Ant System algorithm of Dorigo et al. [1996] has been used for this paper. There are a number of other variations of Ant Colony Optimisation [Stützle and Hoos, 2000]. The water distribution optimisation problem is formulated such that the objective function is based on the cost of the network (pipe purchase costs plus installation costs) and any penalty costs due to the design not meeting design constraints. Typical constraints in the pipe optimisation problem include minimum allowable pressures at demand nodes, maximum allowable pressures at demand nodes and maximum permissible velocities of flow in pipes. For each design considered as part of the ACO process, one or more hydraulic simulations (depending on the number of demand loading cases e.g. peak hour, fire demand loadings and/or extended period simulation) using a computer simulation model to determine flows in all pipes and pressures at all nodes in the water distribution system are conducted. Once the pressures and velocities are known then the results can be checked to find if there are any design constraints that have been violated.

In the Maier et al. [2001] formulation for water distribution system optimisation, one decision point was associated with the selection of the diameter of each pipe that is a decision variable within the water distribution system design. For example, the diameter of each pipe as shown in Figure 1 may need to be determined by optimisation. The decision points are (d_1, d_2, d_3, d_4, d_5). At each decision point there are a number of options corresponding to discrete commercially available pipe sizes. Unit cost data for each of the pipe options must be available.

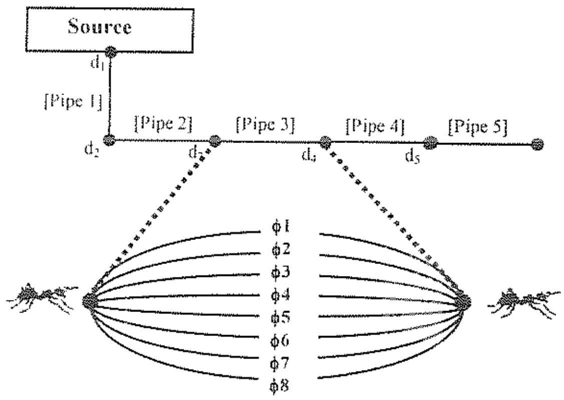


Figure 1. Ant colony optimisation formulation for optimising the design of a water distribution system [Maier et al., 2001].

A population or colony of m ants are used to traverse the network sequentially making decisions at each decision point. The traversal of a particular pipe is referred to as a cycle. Each trial solution is built incrementally as each of the m artificial ants moves from one decision point to the next. Each cycle of the ACO consists of three main steps: generation of m trial solutions by the traversal of the entire pipe network by a population or colony of m ants, calculation of the cost of each of the m trial solutions, and updating of the concentrations of the pheromone trails associated with each of the choices. The pheromone is usually updated only after an entire cycle has been constructed. An ACO run is made up of many cycles. A cycle is equivalent to a generation for genetic algorithm optimisation. To understand details of the parameters involved in the ACO, the basic equations for the operation of the ACO from the Maier et al. [2001] paper are repeated below for completeness.

At a decision point (or at cycle k or for pipe i within a cycle), an ant chooses one of the available pipe choices j based on the following equation [Dorigo et al., 1996, Maier et al., 2001]:

$$p_{i(j)}(k) = \frac{[\tau_{i(j)}(k)]^{\alpha} \cdot [\eta_{i(j)}]^{\beta}}{\sum_{l_{i(j)}} [\tau_{i(j)}(k)]^{\alpha} \cdot [\eta_{i(j)}]^{\beta}} \qquad (1)$$

where $p_{i(j)}$ (k) is the probability that option $l_{i(j)}$ is chosen for pipe i, $\tau_{i(j)}$ (k) is the concentration of pheromone associated with option $l_{i(j)}$ for pipe i, $\eta_{i(j)} = 1/c_{i(j)}$ is the visibility or a heuristic factor favoring options that have smaller "local" pipe costs (this is not modified during a run), $c_{i(j)}$ = cost of an option for a pipe, and α and β are parameters that control the relative importance of pheromone and the local heuristic factor, respectively. In the first cycle of the ACO the selection of pipe options is random, as there is an equal probability that each option will be chosen.

After each cycle, pheromone trails are updated. First, pheromone is evaporated on all pipes based on a pheromone persistence coefficient ρ (which is always less than one). Pheromone is then updated by an amount $\Delta\tau_{i(j)}$ as follows:

$$\tau_{i(j)}(k+1) = \rho\,\tau_{i(j)}(k) + \Delta\tau_{i(j)} \qquad (2)$$

where $\tau_{i(j)}(k+1)$ is the concentration of pheromone associated with option $l_{i(j)}$ at cycle k+1, $\tau_{i(j)}$ (k) is the concentration of pheromone associated with option $l_{i(j)}$ at cycle k. The total change in pheromone $\Delta\tau_{i(j)}$ is given by:

$$\Delta\tau_{i(j)} = \sum_{h=1}^{m} \Delta\tau_{i(j)}^{h} \qquad (3)$$

where $\Delta\tau_{i(j)}^{h}$ is the change in the concentration of pheromone associated with option $l_{i(j)}$ made by ant h between cycles k and k+1 while m = the total number of ants used for each cycle. Note that pheromone updating is based on every ant in the previous cycle. The ant-cycle algorithm was used by Maier et al. [2001] where the pheromone update quantity $\Delta\tau_{i(j)}^{h}$ is given by:

$$\Delta\tau_{i(j)}^{h} = \begin{cases} \dfrac{R}{f(\varphi)^{h}} - P_{pher} * \Delta H_{max} & \text{if the } h^{th} \text{ ant chooses option } l_{i(j)} \text{ at cycle } k \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

where R is the pheromone reward factor, $f(\varphi)^{h}$ is the cost of the trial solution generated by ant h, P_{pher} is a pheromone penalty factor and ΔH_{max} is the maximum pressure deficit from the hydraulic simulation of the network. Solution pipe component options that are used by many ants and form part of lower cost solutions receive more pheromone and are more likely to be chosen in future cycles [Stützle and Hoos, 2000].

3. SENSITIVITY OF PERFORMANCE OF THE ACO FOR A CASE STUDY

3. 1 The Case Study

The case study used is a two reservoir 14-pipe network presented by Simpson et al. [1994] as shown in Figure 2. There are 5 new pipes (a minimum sized pipe must be incorporated at the very least) and 3 pipes that can be duplicated or cleaned if required. There are 3 demand loading cases: a peak hour and two fire loading cases. There are 8 options or choices for each pipe ranging from 152 to 509mm. The search space size is 32.768 million possible combinations. A complete enumeration was performed by Murphy and Simpson [1992] to find the global optimal solution of $1.750 million. Note that usually the global optimum solution would not be known.

3. 2 The ACO Parameters

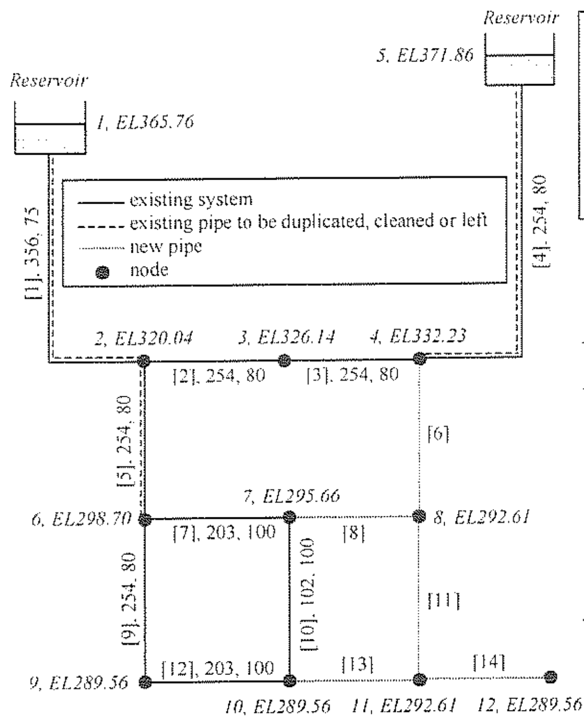
The parameters that can be chosen for an ACO run are shown in Table 1. Default values used for this case study investigation are also given.

3. 3 The α and β Parameters

The effects of different combinations of α and β values on the minimum cost obtained for the 14-pipe network problem are investigated. A range of α values that control the relative importance of the pheromone trail concentrations were tested while β was varied. For $\alpha < 1$ the global optimum was not found. Only when α was increased to 3 was a substantial number of global optima found.

From Table 2, it appears that the best value of α is in the range from 3.5 to 5.0. This was also found for a 6-pipe problem.

The results for different combinations of α and β are plotted in Figure 3 to determine the relationship between these two parameters in the algorithm. Default values in Table 1 are used for the other parameters. From Figure 3 it is evident that if the heuristic factor favoring options that have smaller "local" pipe costs (β) is outside the range (1.0 to 3.5) then the global optimum is rarely found. The best value of β appears to be 3.0. Similar results were also found for a 6-pipe problem and by Dorigo et al. [1991]. When β exceeds 3.5 the algorithm focussed too much on the lower cost pipe options and is driven too much by the visibility. The pressure penalties are not given enough weight.



[1], 356, 75 [pipe number], diameter (mm), present day Hazen-Williams roughness coefficient (C)

2, EL320.04 node number, node elevation (m)

Note: 1. All pipe lengths are 1609m, except pipe [1] and pipe [4] which are 4828m and 6437m, respectively.
2. C is 120 for new and cleaned pipes.

	Demand Pattern 1		Demand Pattern 2		Demand Pattern 3	
Node	Mean Q (L/s)	H_{min} (m)	Mean Q (L/s)	H_{min} (m)	Mean Q (L/s)	H_{min} (m)
2	12.62	28.18	12.62	14.09	12.62	14.09
3	12.62	17.61	12.62	14.09	12.62	14.09
4	0	17.61	0	14.09	0	14.09
6	18.93	35.22	18.93	14.09	18.93	14.09
7	18.93	35.22	82.03	10.57	18.93	14.09
8	18.93	35.22	18.93	14.09	18.93	14.09
9	12.62	35.22	12.62	14.09	12.62	14.09
10	18.93	35.22	18.93	14.09	18.93	14.09
11	18.93	35.22	18.93	14.09	18.93	14.09
12	12.62	35.22	12.62	14.09	50.48	10.57

Figure 2. Case study – two reservoir 14-pipe network [after Simpson et al., 1994].

Table 1. The ACO parameters.

Parameter (Default)	Description
m (100)	Number of ants in population
ρ (0.8)	pheromone persistence coefficient
α	Parameter controlling the relative importance of pheromone τ for the pipe choice probability – Eq. 1
β	Parameter controlling the relative importance of local heuristic factor η for the pipe choice probability – Eq. 1
R (200,000)	the pheromone reward factor
P_{pher} (0.005)	the pheromone penalty factor

3.4 The ρ Parameter

The pheromone persistence factor ρ controls the evaporation of pheromone from each of the pipes at the end of each cycle. Pheromone evaporation is needed to avoid premature convergence of the ACO to a sub-optimal region of the search space.

Table 2. Sensitivity of ACO performance to α.

α	β values at which optimum was found	Average number of evaluations* to find optimum
1	3, 3.5	25,322
3	1.5, 2.0, 2.5, 3.0	27,432
3.5	1.5, 2.0, 2.5, 3.0	20,473
4	1.5, 2.0, 3.0	19,766
4.5	1.0, 1.5, 2.0, 2.5,	23,551
5	1.0, 1.5, 2.5, 3.0	21,127

* No. of evaluations = No. of ants(m) x Maximum no. of cycles

The higher the evaporation the slower the convergence to the solution. Figure 4 shows the performance of the ACO when different values of ρ are used for 5 different starting random number seeds. For a $\rho < 0.2$ the ACO does not locate the global optimum. It appears that for any value of $\rho > 0.4$ and up to 1.0 the ACO performs well, even for no evaporation ($\rho = 1.0$). A value of $\rho = 0.8$ gave a lesser number of evaluations needed to achieve the minimum cost network. Dorigo et al. [1996] found that a $\rho = 0.99$ was best for the TSP Problem.

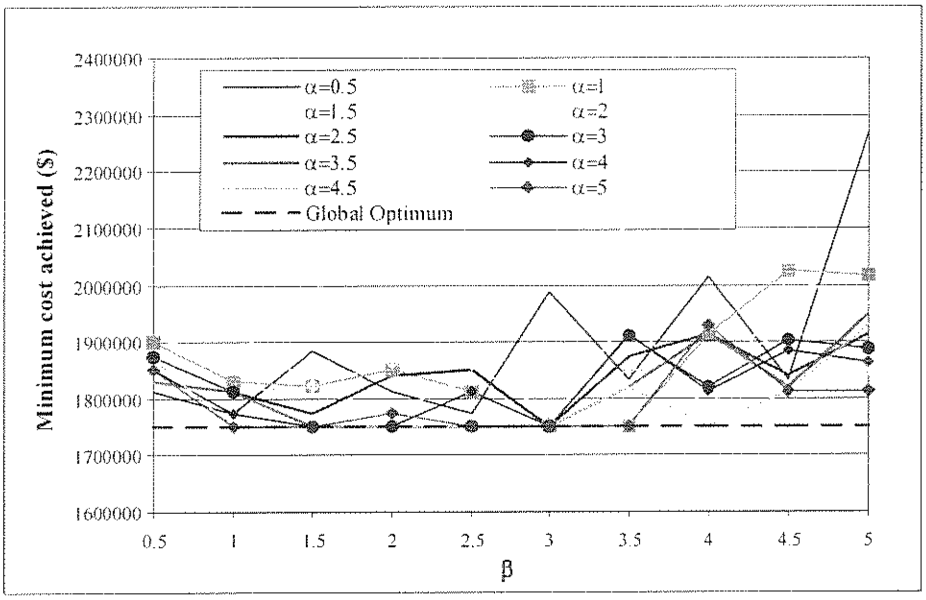


Figure 3. The effect of different combinations of α and β values.

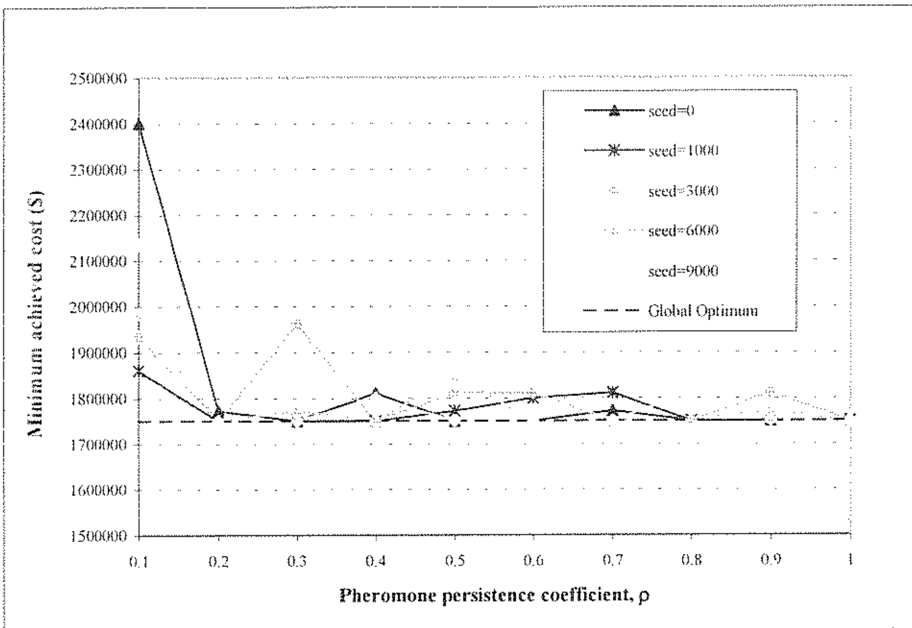


Figure 4. Minimum cost achieved for different pheromone persistence coefficients ρ.

3. 5 The R and P_{pher} Parameters

Reward and penalty are two mechanisms that alter the pheromone concentrations for each of the options at each of the decision nodes. Positive reinforcement occurs to the pipe diameter options that result in solutions of high fitness (via the R factor) and negative reinforcement to pipe diameter options that result in solutions that violate the pressure constraints (via the P factor) in Eq. (4).

Penalty factors in the range of 0.001 to 0.01 were tested for seven different starting random number seeds for a reward factor of R = 200,000. The best performance occurred for the range P = (0.004, 0.01). Clearly there is a relationship between R and P. Table 3 shows the best P values found for a range of R values.

Table 3. Optimal P values for reward R values.

Reward, R	Penalty Factor, P
50,000	approximately 0.0015
100,000	0.002 ~ 0.003
200,000	0.004 ~ 0.007
500,000	0.01~0.02

3. 6 The Number of Ants m

A range of m =1 to 5000 ants was tested (1, 2, 5, 10, 50, 100, 150, 200, 300, 400, 500, 800, 900, 1000, 2000, 3000, 4000 and 5000). It was found that the performance of the ACO was similar for a range of m = (50, 800). Pheromone updating is only carried out after each cycle of m ants have traversed the network.

4. CONCLUSIONS

The Ant Colony Optimisation (ACO) method for optimising the design of water distribution systems has been considered in this paper. The formulation for the problem has been presented in earlier papers. The focus of this paper has been to investigate the sensitivity of the performance of the ACO method to changes in parameters. There are 6 parameters that need to be selected: (1) the number of ants in a population m, (2) the parameter for controlling the relative importance of pheromone τ for the pipe choice probability α, (3) the parameter for controlling the relative importance of the local heuristic factor η for the pipe choice probability β, (4) the pheromone persistence coefficient ρ (5) the pheromone reward factor R and (6) the pheromone penalty factor P_{pher}. A case study network comprising 14-pipes has been optimised using ACO for different combinations of parameters. The global optimum solution has been previously determined so it is easy to assess if ACO is finding the global optimum solution. Optimal ranges for all of these parameters are recommended as an outcome of these investigations (α =3.0 to 5.0, β = 3.0, m = 50 to 800, ρ = 0.8, R and P – see Table 3). More work is needed to better assess the inter-relationship between the pheromone reward factor R and the penalty factor P_{pher}. As with other evolutionary optimisation methods, parameters need to be selected. Appropriate values for these parameters will initially be gained from experience of users but an improved basis needs to be developed. This should be the focus of further research.

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